

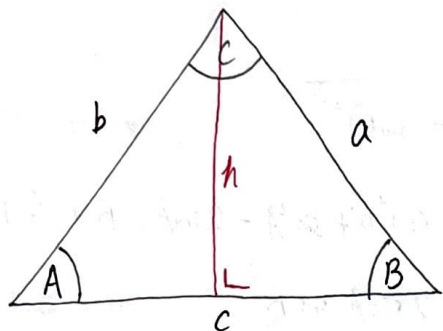
* Trigonometric Identities:

• Identity 2 (Sine Rule)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Derivation: There are two cases to consider; A is acute or A is obtuse.

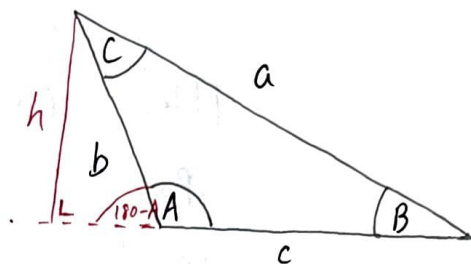
Case 1: A is acute.



$$\sin A = \frac{h}{b} \Rightarrow h = b \sin A$$

$$\sin B = \frac{h}{a} \Rightarrow h = a \sin B$$

Case 2: A is obtuse.



$$\sin(180^\circ - A) = \frac{h}{b} \Rightarrow \sin A = \frac{h}{b} \Rightarrow h = b \sin A$$

$$\sin B = \frac{h}{a} \Rightarrow h = a \sin B$$

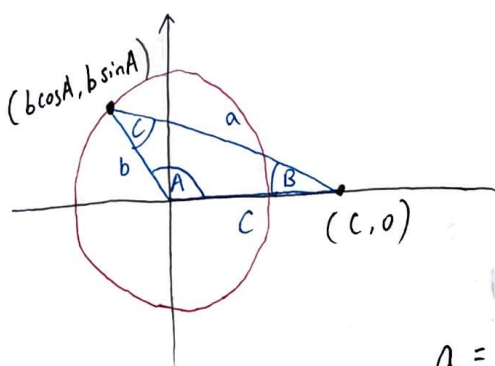
Same equations

$$a \sin B = b \sin A \Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\text{Similarly } \frac{a}{\sin A} = \frac{c}{\sin C}, \therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{Q.E.D.}$$

• Identity 3 (Cosine Rule)

$$a^2 = b^2 + c^2 - 2bc \cos A$$



Derivation: Construct a circle of radius b and centre at (0,0). Place a triangle with angle A at (0,0) and side c along x-axis.

Find distance between (b cos A, b sin A) and (c, 0)

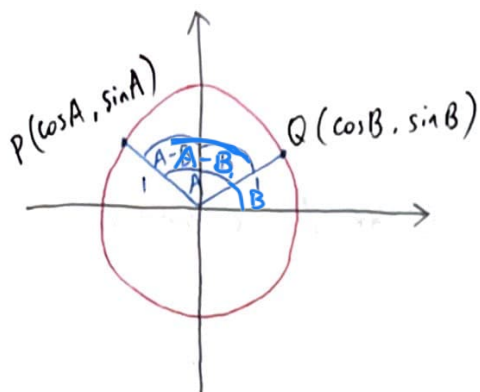
$$a = \sqrt{(b \cos A - c)^2 + (b \sin A - 0)^2} \quad \dots \text{distance between point}$$

$$a^2 = b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A \quad \dots \text{square both side}$$

$$a^2 = b^2 (\cos^2 A + \sin^2 A) + c^2 - 2bc \cos A$$

Q.E.D.

Identity 4: $\cos(A-B) = \cos A \cos B + \sin A \sin B$.



Derivation:

$P(\cos A, \sin A)$ and $Q(\cos B, \sin B)$ are points on unit circle.

Using Cosine Rule:

$$|PQ|^2 = 1^2 + 1^2 - 2(1)(1)\cos(A-B)$$

$$|PQ|^2 = 2 - 2\cos(A-B)$$

Using distance formula:

$$|PQ| = \sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2} \quad \dots \text{square both side.}$$

$$|PQ|^2 = \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$$

$$|PQ|^2 = 2 - 2\cos A \cos B - 2\sin A \sin B$$

Let two ^(expression) equations equal to each other:

$$2 - 2\cos(A-B) = 2 - 2\cos A \cos B - 2\sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \quad \text{Q.E.D.}$$

Identity 5: $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

Derivation: $\cos(A+B) = \cos(A - (-B))$

$$= \cos A \cos(-B) + \sin A \sin(-B) \quad \dots \text{Id 4.}$$

$$= \cos A \cos B - \sin A \sin B \quad \text{Q.E.D.} \quad \dots \text{Negative angles}$$

Identity 6: $\cos 2A = \cos^2 A - \sin^2 A$

Derivation: $\cos 2A = \cos(A+A)$

$$= \cos A \cos A - \sin A \sin A \quad \dots \text{Id 5}$$

$$= \cos^2 A - \sin^2 A$$

Identity 7: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Derivation: $\sin(A+B) = \cos(90^\circ - (A+B))$... Complementary angles
 $= \cos((90^\circ - A) - B)$
 $= \cos(90^\circ - A) \cos B + \sin(90^\circ - A) \sin B$... Id 4
 $= \sin A \cos B + \cos A \sin B$ Q.E.D.

Identity 8: $\sin(A-B) = \sin A \cos B - \cos A \sin B$

skip this proof because it's the same approach as $\cos(A+B)$
 (Don't need to learn) i.e use $\sin(A+(-B))$

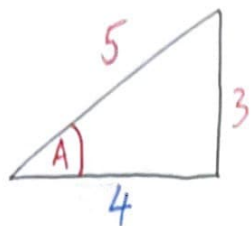
Identity 9: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Derivation: $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$
 $= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$... Id 7
 ... Id 5
 $= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$... dividing above and below by $\cos A \cos B$
 $= \frac{\tan A + \tan B}{1 - \tan A \tan B}$ Q.E.D.

* Using trig ratios and trig identities

If question only gives $\sin A = \frac{3}{5}$, then you can work out others by yourself

i.e.



$$\therefore 3^2 + x^2 = 5^2 \quad \therefore \cos A = \frac{4}{5}$$

$$x = 4 \quad \text{and} \quad \tan A = \frac{3}{4}$$

We can then also work out $\sin 2A$, $\cos 2A$ or $\tan 2A$ and so on.

$$\sin 2A = 2 \sin A \cos A = 2 \left(\frac{3}{5} \right) \left(\frac{4}{5} \right) = \frac{24}{25}$$

$$\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{4}{5} \right)^2 - \left(\frac{3}{5} \right)^2 = \frac{7}{25}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \left(\frac{3}{4} \right)}{1 - \left(\frac{3}{4} \right)^2} = \frac{24}{7}$$

* Trig Application Useful formulas: (Most in log table but worth to memorise).

Arc length

$$l = r\theta, \quad \dots \theta \text{ in radians}$$

$$l = \frac{\theta}{180} \pi r, \quad \dots \theta \text{ in degrees}$$

Area of Sector

$$A = \frac{1}{2} r^2 \theta, \quad \dots \theta \text{ in radians}$$

$$A = \frac{\theta}{360} \pi r^2, \quad \dots \theta \text{ in degrees}$$

Area of a triangle

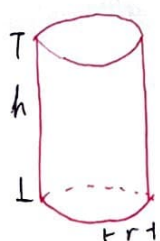
$$A = \frac{1}{2} (\text{base}) (\perp \text{ height})$$

$$A = \frac{1}{2} ab \sin C$$

Sine ~~Rule~~ Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$

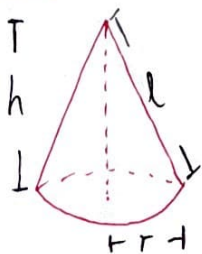
$$A = \cos^{-1} \left[\frac{b^2 + c^2 - a^2}{2bc} \right]$$



Cylinder:

$$A = 2\pi rh$$

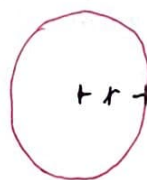
$$V = \pi r^2 h$$



Cone:

$$A = \pi r l$$

$$V = \frac{1}{3} \pi r^2 h$$



Sphere:

$$A = 4\pi r^2$$

$$V = \frac{4}{3} \pi r^3$$