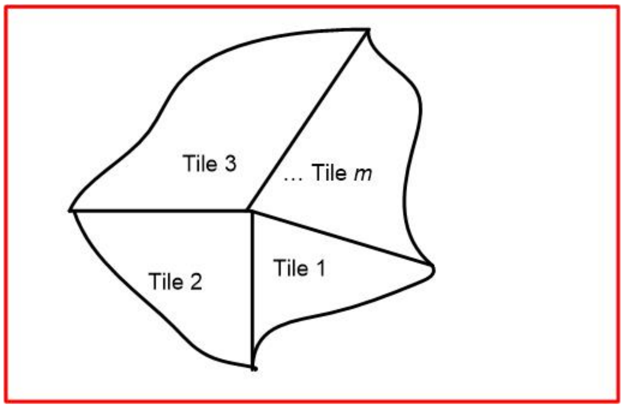
So what shapes can we fit together?

(...without any tiles overlapping or leaving gaps...)



- Imagine tiles in the shape of a regular n-sided polygon. If they do fit together without leaving gaps and without overlapping, then they must fit together as in the diagram above, with m tiles all meeting at a point.

- m and n are not necessarily equal; and they must (of course!) be whole numbers.

In the diagram above, we are showing the interior angles of all m tiles. It is obvious that all such angles must add up to 360^o.

So if we call the interior angles x, then

$$mx = 360^o.$$

If each polygon has n sides, then the interior angles of each polygon must add to $180(n-2)^o$ (we discussed this result today, 2 March).

This means that each interior angle must be $180(n-2)/n$ degrees.

If m of these tiles fit together (see the above diagram), then m such interior angles must add up to 360^o. So:

$$\frac{180m(n-2)}{n} = 360^o$$

which can be rearranged to give

$$m = \frac{2n}{(n-2)}.$$

Remember that m is the number of **tiles** which meet at a point, and n is the number of **sides of each polygonal tile**.

So if we treat n as the independent variable and m as the dependent variable, we can draw up a table, and use the equation

$$m = \frac{2n}{(n-2)}$$

to calculate m for any value of n, as follows:

n	m
3	6
4	4
5	10/3
6	3
7	14/5
8	8/3
9	18/7
10	5/2
11	22/9
12	12/5
13	26/11
etc	

This tells us, for example, that if we fit 6-sided polygons together, then three of them will meet at one point. If on the other hand, we try and fit 5-sided polygons together, we find that "10/3" will fit at a point: in other words there will be overlaps.

As you can see, there are only three solutions in which both n and m are whole numbers: $n = 3, 4$ and 6. In other words, only triangles, squares and hexagons will fit together. Nothing else will!